AMC 10/12 Student Practice Questions

You will find these and additional problems for the AMC 10 and AMC 12 on AMC's web site: amc.maa.org, available from the current and previous AMC 10/12 Teacher Manuals, amc.maa.org/amc/e-exams/e6-amc12/archive12.shtml) or from our Problems page archives (amc.maa.org/amc/a-activities/a7-problems/problem81012archive.shtml).

A cell phone plan costs 20 dollars each month, plus 5 cents per text message sent, plus 10 cents for each minute used over 30 hours. In January Michelle sent 100 text messages and talked for 30.5 hours. How much did she have to pay?

(A) $\$24.00$ **(B)** $\$24.50$ **(C)** $\$25.50$ **(D)** $\$28.00$ **(E)** $\$30.00$

2011 AMC 10A, Problem #1—
2011 AMC 12A, Problem #1—
"How many minutes of excess chatting are there, and what do they cost?"

Solution

Answer (D): The text messages cost $\$0.05 \cdot 100 = \$5.00$, and the 30 minutes of excess chatting cost $\$0.10 \cdot 30 = \$3.00$. Therefore the total bill came to $\$5 + \$3 + \$20 = \$28$.

Difficulty: Easy

CCSS-M: N-Q.1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas.

Let X and Y be the following sums of arithmetic sequences:

$$X = 10 + 12 + 14 + \cdots + 100,$$
$$Y = 12 + 14 + 16 + \cdots + 102.$$

What is the value of $Y - X$?

(A) 92 **(B)** 98 **(C)** 100 **(D)** 102 **(E)** 112

2011 AMC 10A, Problem #4—
"What are the elements in Y but not in X, and vice versa?"

Solution

Answer (A): Every term in X except 10 appears in Y. Every term in Y except 102 appears in X. Therefore $Y - X = 102 - 10 = 92$.

OR

The sum X has 46 terms because it includes all 50 even positive integers less than or equal to 100 except for 2, 4, 6, and 8. The sum Y has the same number of terms, and every term in Y exceeds the corresponding term in X by 2. Therefore $Y - X = 46 \cdot 2 = 92$.

Difficulty: Medium
CCSS-M: A-SSE.2. Use the structure of an expression to identify ways to rewrite it.

Square $EFGH$ has one vertex on each side of square $ABCD$. Point E is on $\overline{AB}$ with $AE = 7 \cdot EB$. What is the ratio of the area of $EFGH$ to the area of $ABCD$?

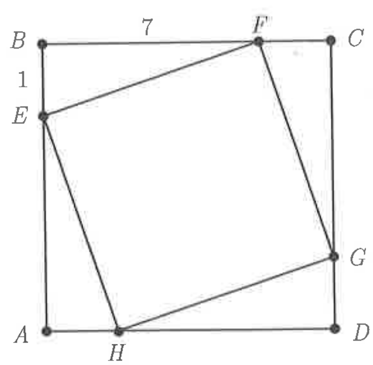
(A) $\frac{49}{64}$ **(B)** $\frac{25}{32}$ **(C)** $\frac{7}{8}$ **(D)** $\frac{5\sqrt{2}}{8}$ **(E)** $\frac{\sqrt{14}}{4}$

2011 AMC 10A, Problem #11— "What is the length of EB?"

Solution

Answer (B): Without loss of generality, assume that F lies on $\overline{BC}$ and that $EB = 1$. Then $AE = 7$ and $AB = 8$. Because $EFGH$ is a square, $BF = AE = 7$, so the hypotenuse $\overline{EF}$ of $\triangle EBF$ has length $\sqrt{1^2 + 7^2} = \sqrt{50}$. The ratio of the area of $EFGH$ to that of $ABCD$ is therefore

$$\frac{EF^2}{AB^2} = \frac{50}{64} = \frac{25}{32}.$$



Difficulty: Medium Hard
CCSS-M: G-GPE.6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Josanna's test scores to date are 90, 80, 70, 60, and 85. Her goal is to raise her test average at least 3 points with her next test. What is the minimum test score she would need to accomplish this goal?

(A) 80 **(B)** 82 **(C)** 85 **(D)** 90 **(E)** 95

2011 AMC 10B, Problem #2—
2011 AMC 12B, Problem #2—
"What is the sum of her first five test scores?"

Solution

Answer (E): The sum of her first 5 test scores is 385, yielding an average of 77. To raise her average to 80, her 6^{th} test score must be the difference between $6 \cdot 80 = 480$ and 385, which is 95.

Difficulty: Medium Easy
CCSS-M: A-CED.1. Create equations and inequalities in one variable and use them to solve problems.

The sum of two angles of a triangle is $\frac{6}{5}$ of a right angle, and one of these two angles is 30° larger than the other. What is the degree measure of the largest angle in the triangle?

(A) 69 **(B)** 72 **(C)** 90 **(D)** 102 **(E)** 108

2011 AMC 10B, Problem #7—
"What are the degree measures of the two angles?"

Solution

Answer (B): The degree measures of two of the angles have a sum of $\frac{6}{5} \cdot 90 = 108$ and a positive difference of 30, so their measures are 69 and 39. The remaining angle has a degree measure of $180 - 108 = 72$, which is the largest angle.

Difficulty: Medium 8.G.5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles,
CCSS-M:

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Consider the set of numbers $\{1, 10, 10^2, 10^3, \ldots, 10^{10}\}$. The ratio of the largest element of the set to the sum of the other ten elements of the set is closest to which integer?

(A) 1 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 101

2011 AMC 10B, Problem #10—
"What is the sum of the smallest ten elements?"

Solution

Answer (B): The sum of the smallest ten elements is

$$1 + 10 + 100 + \cdots + 1{,}000{,}000{,}000 = 1{,}111{,}111{,}111.$$

Hence the desired ratio is

$$\frac{10{,}000{,}000{,}000}{1{,}111{,}111{,}111} = \frac{9{,}999{,}999{,}999 + 1}{1{,}111{,}111{,}111} = 9 + \frac{1}{1{,}111{,}111{,}111} \approx 9.$$

OR

The sum of a finite geometric series of the form $a(1+r+r^2+\cdots+r^n)$ is $\frac{a}{1-r}(1-r^{n+1})$. The desired denominator $1 + 10 + 10^2 + \cdots + 10^9$ is a finite geometric series with $a = 1, r = 10$, and $n = 9$. Therefore the ratio is

$$\frac{10^{10}}{1 + 10 + 10^2 + \cdots + 10^9} = \frac{10^{10}}{\frac{1}{1-10}(1 - 10^{10})} = \frac{10^{10}}{10^{10} - 1} \cdot 9 \approx \frac{10^{10}}{10^{10}} \cdot 9 = 9.$$

Difficulty: Medium

CCSS-M: A-SSE.4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

A power boat and a raft both left dock A on a river and headed downstream. The raft drifted at the speed of the river current. The power boat maintained a constant speed with respect to the river. The power boat reached dock B downriver, then immediately turned and traveled back upriver. It eventually met the raft on the river 9 hours after leaving dock A. How many hours did it take the power boat to go from A to B?

(A) 3 **(B)** 3.5 **(C)** 4 **(D)** 4.5 **(E)** 5

2011 AMC 12A, Problem #12—

"Consider the speed of the river and the speed of the power boat upstream and downstream."

Solution

Answer (D): Assume the power boat and raft met at point O on the river. Let x be the speed of the boat and y be the speed of the raft and the river current. Then $x + y$ is the speed of the power boat downstream and $x - y$ is the speed of the power boat upstream. Let the distance AB between the docks be S, so that $AO = 9y$ and $OB = S - 9y$. Then because time is equal to distance divided by rate,

$$\frac{S}{x+y} + \frac{S-9y}{x-y} = 9.$$

Rearrange to find that $S = \frac{9}{2}(x+y)$. Then the time it took the power boat to go from A to B is

$$\frac{S}{x+y} = \frac{9(x+y)}{2(x+y)} = 4.5.$$

OR

In the reference frame of the raft, the boat simply went away, turned around, and came back, all at the same speed. Because the trip took 9 hours, the boat must have turned around after 4.5 hours.

Difficulty: Medium Easy

CCSS-M: A-CED.2. Create equations in two or more variables to represent relationships between quantities.

Suppose that $|x+y|+|x-y|=2$. What is the maximum possible value of x^2-6x+y^2 ?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

2011 AMC 12A, Problem #18—
"What are the graph formed by the expressions?"

Solution

Answer (D): The graph of the equation $|x+y|+|x-y|=2$ is a square formed by the lines $x=\pm 1$ and $y=\pm 1$. For $c>-9$, the equation $c=x^2-6x+y^2=(x-3)^2+y^2-9$ is the equation of a circle with center $(3,0)$ and radius $\sqrt{c+9}$. Among all such circles that intersect the square, the largest one contains the points $(-1,\pm 1)$ and has radius $\sqrt{4^2+1^2}=\sqrt{17}$. It follows that the maximum value of c is $17-9=8$.

Difficulty: Hard

CCSS-M: G-GPE.1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

A frog located at (x, y), with both x and y integers, makes successive jumps of length 5 and always lands on points with integer coordinates. Suppose that the frog starts at $(0,0)$ and ends at $(1,0)$. What is the smallest possible number of jumps the frog makes?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

2011 AMC 12B, Problem #11— "What is the smallest number of jumps?"

Solution

Answer (B): Because $AB = 1$, the smallest number of jumps is at least 2. The perpendicular bisector of $\overline{AB}$ is the line with equation $x = \frac{1}{2}$, which has no points with integer coordinates, so 2 jumps are not possible. A sequence of 3 jumps is possible; one such sequence is $(0,0)$ to $(3,4)$ to $(6,0)$ to $(1,0)$.

Difficulty: Medium Hard
CCSS-M: G-GPE.4. Use coordinates to prove simple geometric theorems algebraically.

How many positive two-digit integers are factors of $2^{24} - 1$?

(A) 4 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

2011 AMC 12B, Problem #15— "Factor the expression until it results in a product of primes."

Solution

Answer (D): Factoring results in the following product of primes:

$$\begin{aligned} 2^{24} - 1 &= (2^{12} - 1)(2^{12} + 1) = (2^6 - 1)(2^6 + 1)(2^4 + 1)(2^8 - 2^4 + 1) \\ &= 63 \cdot 65 \cdot 17 \cdot 241 = 3 \cdot 3 \cdot 5 \cdot 7 \cdot 13 \cdot 17 \cdot 241. \end{aligned}$$

The two-digit integers that can be formed from these prime factors are:

$$\begin{gathered} 17, \quad 3 \cdot 17 = 51, \quad 5 \cdot 17 = 85, \\ 13, \quad 3 \cdot 13 = 39, \quad 5 \cdot 13 = 65, \quad 7 \cdot 13 = 91, \\ 3 \cdot 7 = 21, \quad 5 \cdot 7 = 35, \quad 3 \cdot 3 \cdot 7 = 63, \\ 3 \cdot 5 = 15, \quad \text{and} \quad 3 \cdot 3 \cdot 5 = 45. \end{gathered}$$

Thus there are 12 positive two-digit factors.

Difficulty: Hard

CCSS-M: A-SSE.1. Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.